Instability of plasma vibrations in a wide parabolic GaAs/AlGaAs quantum well caused by plasmon-assisted tunnelling. A possible mechanism for generation and amplification of far-infrared radiation

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Instability of plasma vibrations in a wide parabolic GaAs/AlGaAs quantum well caused by plasmon-assisted tunnelling. A possible mechanism for generation and amplification of far-infrared radiation

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Abstract. We consider a wide parabolic GaAs/AlGaAs quantum well placed between two metal plates so that direct current can flow across the well. Sloshing plasma oscillations of the charge stored in the well modulate the direct current and charges on the plates and in the well acquire an alternating component. The system can be considered as a capacitor with one plate, the electronic slab in the well, performing mechanical oscillations (plasmons). The electrostatic forces acting between the plates of this capacitor cause the oscillations to build up. We derive an expression for the threshold and estimate the power of the infrared radiation which can be delivered by the structure.

1. Introduction

Direct current-driven plasma instabilities in GaAs/AlGaAs artificial heterostructures (two-dimensional layers and superlattices) have attracted much attention due to the interesting fundamental physics involved and possible applications for generation and amplification of infrared radiation (see for example [1–7]). In particular much work has been done on the interaction of direct current flowing along a two-dimensional layer with plasma waves in the same (or a neighbouring) layer [1–6]. This interaction can result in the growth of the plasma wave magnitude due to the energy transfer from direct current to plasma waves. With a suitable coupling arrangement such as a grating, this energy can be converted to electromagnetic radiation. The basic idea is the same as for the gaseous plasmas in which the current-excited plasma waves have been studied in detail [8]. In the solid state case the observation of the instability is hindered by the fact that the electron velocity is generally smaller than the two-dimensional plasmon velocity. Nevertheless this issue is being pursued both theoretically and experimentally.

In this paper we describe the novel instability mechanism of plasma vibrations in GaAs/AlGaAs heterostructures. Figure 1 shows a schematic diagram of the structure. The main component of the structure is a remotely doped wide parabolic quantum well (PQW) placed between two electrodes (the cathode and the anode). A PQW (proposed by Halperin and Gossard in [9]) produces a thick (> 1000 Å) high-mobility nearly three-dimensional electron slab. The Al composition in the well is graded in such a way as to result in a

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quadratic dependence of the conduction band edge on the distance from the well centre. The electrons in the well screen the quadratic potential and as a result the electron density (n_0) is nearly constant over the thickness of the electron slab and its magnitude is determined by the curvature of the confining parabolic potential. For a wide well filled with a dense electron gas, the electrostatic energy (V_0, figure 1) is much larger than the kinetic energy (V_F, figure 1). The typical investigated areal carrier density (n_s) in the PQW ranges from 10^{11} to 5×10^{11} cm^{-2} [10–13], which corresponds to the three-dimensional density $n_0 = n_s/W$ changing from 10^{16} to 5×10^{16} cm^{-3} for a well width $W = 1000$ Å. The low-temperature mobility in such structures has been reported to be as high as $\mu = 2.5 \times 10^5$ cm^2 V^{-1} s^{-1} [10].

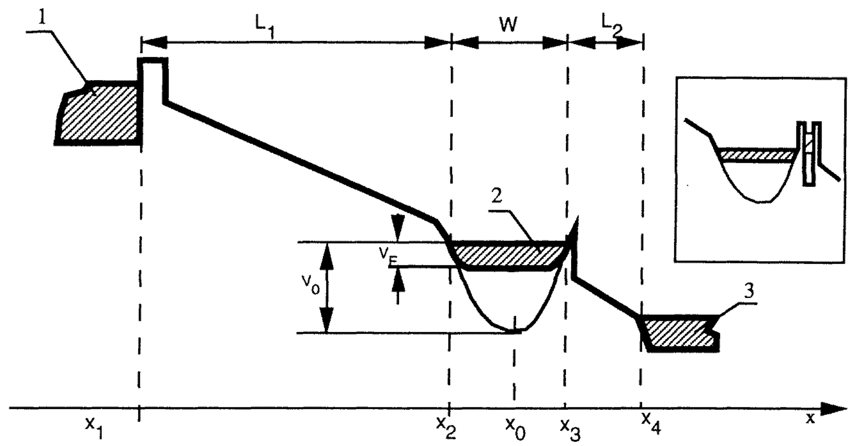

Figure 1. A band-bending diagram of the structure: 1, the cathode; 2, the parabolic quantum well with a triangular tunnel barrier (this well is remotely doped from the cathode side; an unscreened parabolic built-in potential is shown by the thin line); and 3, the anode. Inset: the quantum well with a resonant tunnel barrier.

The electron slab in a PQW can vibrate around the equilibrium position (the sloshing plasma mode) as a whole without changing the charge distribution inside the slab. For this reason the electron–electron interaction plays no role in the plasma oscillations and the plasmon resonant frequency ω_0 is determined by the external parabolic build-in potential [14, 15]. By construction of a PQW the resonant frequency ω_0 coincides with the plasma frequency of a three-dimensional uniform electron gas with density n_0:

$$\omega_0 = \omega_{pl} = \left(\frac{4\pi e^2 n_0}{\varepsilon m}\right)^{1/2}$$

[11], where m is the electron effective mass, ε is the static dielectric constant of the medium and e is the electron charge.

When a DC voltage is applied between the cathode and the anode (figure 1) direct current can flow across the well. We assume that charge leaves the well by tunnelling through the right-hand side barrier of the PQW. The current is assumed to be small in the sense that the plasma vibrations in the well (the sloshing mode) are not strongly affected by it. This requires that the charge transferred by the direct current during one period of plasma vibrations is much smaller than that stored in the well:

$$I_0 T \ll Q_0 \qquad \text{or} \qquad I_0 \ll Q_0 \frac{\omega_0}{2\pi}. \tag{1}$$

Here I_0 is the direct current density flowing across the PQW, T is the period of oscillations and Q_0 is the electric charge (per unit area) in the PQW. The actual values of I_0 satisfying the equation (1) could be quite large. For electron density $n_0 = 5 \times 10^{16}$ cm^{-3}, $\omega_0 = 2\pi \times 2 \times 10^{12}$ s^{-1} and from equation 1 it follows that $I_0 \ll 10^5$ A cm^{-2}. We assume that the electric field between the electrodes is sufficiently high for electrons to move with saturation velocity v_s outside the well.

In this paper we mainly concentrate on the case $L_1 \gg L_2$ which as we shall show later is favourable for observation of the instability. In this case the electron layer in the PQW and the anode constitute a capacitor with one plate (the layer) able to perform mechanical oscillations (plasmons). Mechanical vibrations of the electron layer in the well result in oscillations of the chemical potential of the electrons near the barrier and therefore cause an oscillating component to appear in the electron flow out of the well. This mechanism of the direct current modulation is referred to as plasmon-assisted tunnelling. The modulation of the electron flow causes the amount of charge stored in the PQW to oscillate. This additional oscillating charge which the PQW acquires due to the modulation of direct current induces an opposite charge of nearly the same magnitude on the anode whereas the oscillating charge induced on the cathode and the alternating electric field between the cathode and the PQW are negligible. The oscillating charge on the anode acts on the electronic layer in the PQW with a force, therefore providing a feedback. We will show that this feedback is positive and, if the intrinsic damping of the plasma mode is small, then the feedback will result in the building up of plasma vibrations until the system reaches the stable regime due to some nonlinear mechanism. In this stable regime the alternating field and alternating conductivity current between the PQW and the anode can be quite large whereas outside this region the alternating quantities are small. A consequence of this is that the unavoidable resistances of the contacts (the anode and the cathode) needed for direct current to flow do not influence the instability condition. This makes the $L_1 \gg L_2$ geometry favourable for experimental observation of the instability.

The threshold for the instability depends on the plasma mode damping and the effectiveness of the plasmon-assisted tunnelling. Two-dimensional lateral plasmon-assisted tunnelling from a narrow quantum well is discussed in [16] and is shown to be a rather effective mechanism. Because our system differs from that considered in [16] we cannot directly use the developed formalism in order to estimate plasmon-assisted tunnelling. In this paper our main objective is to describe the physical mechanism leading to the plasma instability and we restrict ourselves to a phenomenological description of the plasmon-assisted tunnelling by a parameter α which relates the position of the electron slab in the PQW to the tunnelling current. Accepting a realistic tunnel barrier's parameters we estimate α.

2. The simple model

Let us consider at first a simple model in which the electric field of the space charge outside the well and the alternating conductivity current in the region to the left of the well (figure 1) are neglected. The width (W) of the PQW is assumed to be much smaller than the distances L_1 and L_2 (figure 1) so that the PQW can be represented by a thin charged layer. The layer can oscillate around an equilibrium position with some resonant frequency ω_0 and this movement affects the charge flow out of the layer. Although all the simplifying assumptions of this model will be lifted in the next section of the paper we would like to stress that if $L_1 \gg L_2$ the simple model leads to the correct equation for the instability threshold. The physical reason for this is that if $L_1 \gg L_2$ the alternating electric field in the region between the cathode and the PQW is much weaker than that in the region between the PQW and the anode. Therefore the conductivity current and space charge are small in this region. What is less obvious and will be shown later is that under the condition $L_1 \gg L_2$ taking account of the space charge between the PQW and the anode will not change the instability threshold either.

Potential differences between the cathode and the PQW and the PQW and the anode are, respectively, $U_1 = E_1 x_w(t)$ and $U_2 = E_2(L - x_w(t))$. Here E_1 and E_2 are x projections of the electric field in the regions $x < x_w$ and $x_w < x < L$ in figure 1, $x_w(t)$ is the instantaneous co-ordinate of the charged layer, and $L = L_1 + L_2$. The potential difference between the cathode and the anode is $U = U_1 + U_2 = E_1 L + (4\pi/\varepsilon)Q(L - x_w(t))$ where Q is the areal charge density ($Q < 0$ for the electronic PQW) of the PQW and the relation $E_2 = E_1 + (4\pi/\varepsilon)Q$ between E_1 and E_2 has been used. The deviations of the quantities involved from equilibrium values are given by the relations $E_i = E_{i0} + \delta E_i$, $x_w = x_0 + \delta x$, $Q = Q_0 + \delta Q$ and so on. For small deviations one has

$$\delta U = \delta E_1 L + \frac{4\pi}{\varepsilon} L_2 \delta Q - \frac{4\pi}{\varepsilon} Q_0 \delta x \tag{2}$$

$$\delta E_2 = \delta E_1 + \frac{4\pi}{\varepsilon} \delta Q \tag{3}$$

and so on.

The continuity equation relates δQ and currents coming in (δI_{c1}) and out (δI_{c2}) of the PQW as follows:

$$\mathrm{j}\omega \delta Q = \delta I_{c1} - \delta I_{c2} \tag{4}$$

where we have assumed the $\exp(\mathrm{j}\omega t)$ time-dependence of all the deviations. In the simple model δI_{c1} in equation (4) is neglected. Small deviations (δx) of the electronic slab in the PQW from its equilibrium position correspond to the proportional change in the tunnel current δI_{c2}:

$$\delta I_{c2} = \alpha \delta x \tag{5}$$

where the phenomenological parameter α accounts for plasmon-assisted tunnelling. It is convenient to introduce the dimensionless parameter α^* according to the relation

$$\alpha = \alpha^* \frac{\omega Q_0}{W} = \alpha^* \omega e n_0 \tag{6}$$

where the electron charge $e < 0$.

Let us now consider the mechanical movement of the slab in the well. The slab experiences a force from the electric field created by charges on the anode and the cathode. This 'external' (with respect to the layer) electric field (E_{ext}) can be found as follows. The field E_1 is composed of E_{ext} and the field $-(2\pi/\varepsilon)Q$, originating from the charge in the well (the minus sign accounts for the accepted positive direction of the x axis in figure 1) so that $E_1 = E_{ext} - (2\pi/\varepsilon)Q$ and for the deviations one has

$$\delta E_1 = \delta E_{ext} - \frac{2\pi}{\varepsilon} \delta Q. \tag{7}$$

The equation of movement for the charged layer coincides with that for an individual electron in the well, so we have

$$(-\omega^2 + \mathrm{j}\omega\gamma + \omega_0^2)\delta x = \frac{e}{m} \delta E_{ext} \tag{8}$$

where γ accounts for a 'friction' force acting on electrons in the well. The value of γ determines the line width of the plasma mode and can be found from far-infrared transmission measurements or calculated from the experimental value of the mobility as $\gamma^{-1} = \tau = \mu m/e$. From the above equations one has

$$\delta x = \frac{e}{m}\frac{1}{A}\delta E_1 \tag{9}$$

where

$$A = -\omega^2 + \omega_0^2 + j\omega\gamma_{eff} \qquad \gamma_{eff} = \gamma - \frac{\omega_{pl}^2\alpha^*}{2\omega}.$$

From equations (2)–(5) and (9) it follows that

$$\delta U = L\delta E_1 + \mathrm{j}\frac{\omega_{pl}^2\omega_0\alpha^*}{\omega A}L_2\delta E_1 - \frac{\omega_{pl}^2}{A}W\delta E_1. \tag{10}$$

If the external circuit represents zero resistance in some frequency range of interest then the boundary condition $\delta U = 0$ and equation (10) determine the stability of the equilibrium state of the system. Using equation (10) we get

$$0 = A + \frac{L_2}{L}\frac{\mathrm{j}\alpha^*\omega_{pl}^2\omega_0}{\omega} - \frac{W}{L}\omega_{pl}^2. \tag{11}$$

If $L_2 \ll L$ and $L \gg W$ the last two terms on the right-hand side of equation (11) are small. The last one represents the shift in the plasmon resonant frequency due to the interaction with the cathode and the anode. Neglecting these terms one has

$$A = -\omega^2 + \omega_0^2 + \mathrm{j}\omega\left(\gamma - \frac{\omega_{pl}^2\alpha^*}{2\omega}\right) = 0. \tag{12}$$

We see that the stability condition coincides with that for a harmonic oscillator with the damping

$$\gamma_{eff} = \gamma - \frac{\omega_{pl}^2\alpha^*}{2\omega}.$$

Therefore the system is unstable and charge in the PQW will oscillate if

$$\alpha^* > \gamma\frac{2\omega}{\omega_{pl}^2} \approx \frac{2\gamma}{\omega_0}. \tag{13}$$

If the condition (13) is fulfilled then the plasma oscillations build up and the above linear small-signal consideration becomes invalid. The magnitude of stable oscillations is determined by the direct current I_0 or by the possible dependence of α^* on the magnitude of charge oscillations in the PQW. It is convenient to illustrate the time-dependence of the quantities involved graphically without the restrictions of the small-signal approximation. This is shown in figure 2. Figure 2(a) shows the position of the 'centre of mass' of the charge distribution in the PQW versus time. The movement of the charge towards the right-hand side barrier is accompanied by in increase of the tunnelling flow of electrons out of the well (figure 2(b)). The time-dependence of the PQW areal electron density $n_s(t) = n_{s0} + \delta n_s(t)$ is determined by electrons flowing in and out of the well and is shown in figure 2(c). Because we have neglected the AC component δI_{c1} the supply of charge to the well is due to the direct current I_0 while electrons leave the well in bunches (figure 2(b)) resulting in pulse-like character of the current I_{c2}. If $L_1 \gg L_2$ an alternating electric charge on the anode $\delta Q_a(t)$ is nearly equal in magnitude and opposite in sign to the alternating charge in the PQW: $\delta Q_a(t) = -\delta Q(t)$. The force ($\delta F_{ext}$) acting on the charge in the PQW from charge $\delta Q_a(t)$ is

$$\delta F_{ext} = -\frac{2\pi}{\varepsilon}|Q_0|\delta Q_a = -\frac{2\pi}{\varepsilon}|Q_0|\delta Q = \frac{2\pi}{\varepsilon}|Q_0||e|\delta n_s.$$

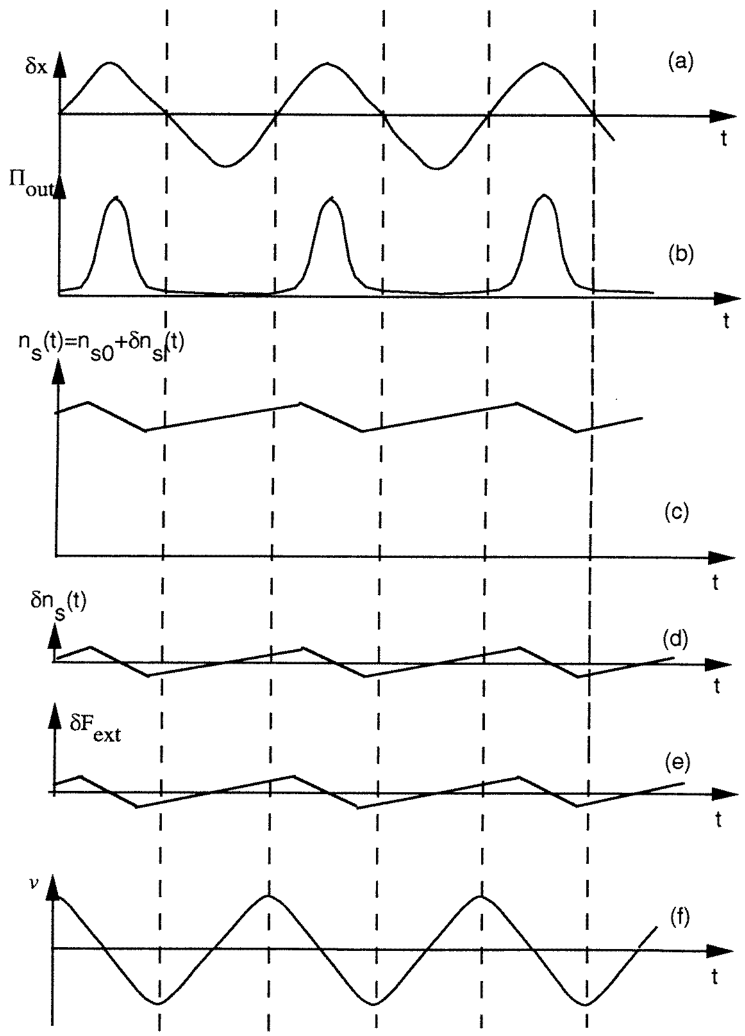

Figure 2. A schematic diagram of the time-dependence of the relevant quantities: (a) the displacement of the electron slab in the well, (b) the electron flow out of the well, (c) the areal electron density in the well, (d) the alternating component of the areal electron density in the well, (e) the alternating component of the external electrostatic force acting on the electron slab in the well and (f) the velocity of the electron slab's movement.

The time-dependence of the force δF_{ext} is shown in figure 2(e). It is seen that δF_{ext} has the same direction as the velocity of the PQW charge (δv, figure 2(f)) and therefore supports the plasma oscillations.

As we have already mentioned, if $L_1 \gg L_2$ the alternating field δE_1 and the conductivity current δI_1 are small. So in the limit $L_2/L_1 \rightarrow 0$ the total alternating current which is the sum of the conductivity and the displacement current is zero:

$$\delta I_{tot} = \delta I_{c1} + \frac{\mathrm{j}\omega\varepsilon}{4\pi}\delta E_1 \approx 0.$$

Because the total current is the same along the circuit it follows that, in the region between the PQW and the anode, the conductivity current compensates for the displacement current:

$$\delta I_{c2} = -\frac{j\omega\varepsilon}{4\pi}\delta E_2$$

while both these currents may be quite large in magnitude. In this case the plasma oscillations in the PQW can be considered to be uncoupled from the external circuit in the sense that there is no alternating current outside the region between the PQW and the anode. The important consequence of this is that contact resistances cannot influence the instability condition (13). Formally this result can be obtained in the following way. If there is some resistance (r) for high-frequency current in the external circuit then the boundary condition $\delta U = 0$ changes to $\delta U + rS\delta I_{tot} = 0$, where S is the area of the structure. Because $\delta I_{c1} \sim \delta E_1$ the total current can be expressed as

$$\delta I_{tot} = \frac{j\omega\tilde{\varepsilon}}{4\pi}\delta E_1$$

where $\tilde{\varepsilon}$ is some proportionality coefficient. Then the boundary condition takes the form

$$\delta U + rS\delta I_{tot} = L\delta E_1 + L_2\frac{\mathrm{j}\alpha^*\omega_{pl}^2\omega_0}{\omega A}\delta E_1 - W\frac{\omega_{pl}^2}{A}\delta E_1 + rS\frac{\mathrm{j}\omega\tilde{\varepsilon}}{4\pi}\delta E_1 = 0$$

or

$$0 = A\left(1 + \frac{rS}{L}\frac{\mathrm{j}\omega\tilde{\varepsilon}}{4\pi}\right) + \frac{L_2}{L}\frac{\mathrm{j}\alpha^*\omega_{pl}^2\omega_0}{\omega} - \frac{W}{L}\omega_{pl}^2. \qquad (14)$$

It is seen that, insofar as the last two terms in equation (14) can be neglected, the stability of the system is still determined by equation (13). The weak coupling of the plasma oscillations with the environment means uneffective transformation of the plasma vibrations into infrared radiation. Nevertheless, if the instability threshold has been reached the magnitude of plasma oscillations can develop to a sufficiently high level to be detected.

To estimate the coefficient α^* one has to specify the tunnelling barrier. We describe the barrier with the help of the current–voltage (I–V) characteristic shown in figure 3(a). The I–V characteristic is assumed to be linear with the current changing from zero to I_m when voltage across the barrier spans some interval Δ/e (figure 3(a)). The parabolic confining potential in the PQW can be expressed in the form $V(x) = (4V_0/W)x^2$ where V_0 is the electrostatic energy of electrons in the PQW as shown in figure 3(b) and the co-ordinate $x = 0$ corresponds to the centre of the PQW (figure 3(b)). The displacement of the charge distribution in the PQW by δx towards the barrier results in an increase in the potential energy of the electrons adjacent to the barrier by $\delta V = (4V_0/W)\delta x$. This in turn increases the current across the barrier by the amount

$$\delta I_{c2} = \frac{I_m}{\Delta}\delta V = \frac{I_m 4V_0}{W\Delta}\delta x$$

so that from equations (5) and (6) it follows that

$$\alpha^* = \frac{4I_m}{\omega Q_0}\frac{V_0}{\Delta}. \qquad (15)$$

The barrier is to be optimized for maximum value of $\alpha^* \sim I_m/\Delta$. A triangular barrier as shown in figure 1 and the resonant tunnel barrier (inset in figure 1) look promising in this respect. We take values $I_m = 10^4$ A cm^{-2}, $\Delta = 10$ meV, $V_0 = 150$ meV [10], $Q_0 = 5\times10^{11}e$. These values are relevant to the tunnelling through an intermediate resonant state in a double-barrier resonant tunnelling structure (RTS). We have chosen this type of tunnelling barrier in order to estimate α^* because it has been investigated experimentally

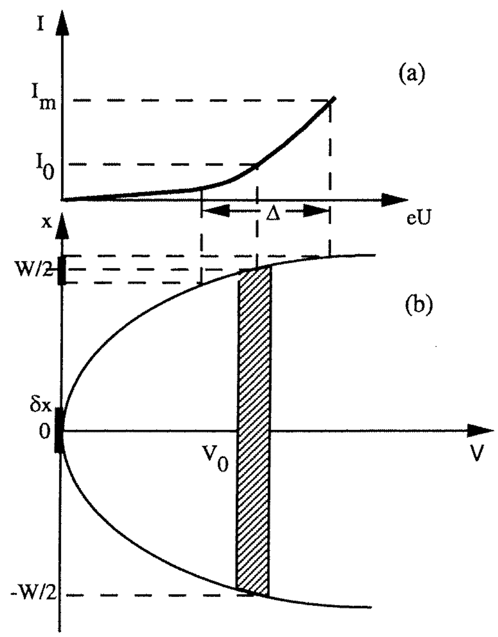

Figure 3. An illustration of the plasmon-assisted tunnelling: the displacement of the electron slab in the well by δx corresponds to a change in electron chemical potential near the barrier by Δ: (a) the I–V characteristic of a tunnel barrier and (b) the band-bending diagram for the electron wide quantum well.

in great detail [17–19]. The value $\Delta = 10$ meV ensures that the quasi-bound-state lifetime τ^* ($\tau^* = \hbar/\Gamma \approx \hbar/\Delta$, Γ is the width at half maximum of the transmission probability function [17, 18]) is of the order of 10^{-13} s, which is much smaller than the period of the oscillations. The required value of τ^* corresponds to a thin ($\leqslant 20$ Å) AlGaAs tunnel barrier [17]. The value of $I_m = 10^4$ A cm^{-2} is based on experimental values of the peak current through a RTS [17–19]. The value $\Gamma = \Delta = 10$ meV implies that the peak current density will be larger than 10^5 A cm^{-2} if the electron density in the cathode of a RTS is 10^{18} cm^{-3}. Therefore, for electron density in the PQW $n_0 = 5 \times 10^{16}$ cm^{-3}, one can take the peak current density to be 10^4 A cm^{-2}. On substituting the above numerical values one gets $\alpha^* = 0.6$ which is significantly larger than $2\gamma/\omega_0 = 0.05$ for the electron mobility in the PQW 10^5 cm^2 V^{-1} s^{-1} and $\omega_0 = 2\pi \times 2 \times 10^{12}$ s^{-1}. There is another dissipative mechanism to consider. Let N be the ratio of Q_0 to the amount of charge crossing the PQW during one period of oscillation. Electrons entering the PQW from the left-hand side (figure 1) have momentum different from that of oscillating electrons in the well. The electron–electron interaction in the well can therefore result in fractional energy losses during one period of at most $2/N$. The quality factor associated with this mechanism of dissipation is πN and the corresponding increase in plasmon line width is $\gamma' = \omega_0/(\pi N)$. For the instability to happen one needs $\alpha^* > 2(\gamma + \gamma')/\omega_0$.

If we take $I_m \approx I_0$, which can be seen from figure 2(b) in figure 2 to be a good approximation then from equation (15) it follows that

$$\alpha^* = \frac{8}{2\pi}\frac{I_0 T}{Q_0}\frac{V_0}{\Delta} \approx \frac{1}{N}\frac{V_0}{\Delta}. \qquad (16)$$

The instability condition can now be re-written in the form

$$\frac{1}{N}\frac{V_0}{\Delta} > \frac{2\gamma}{\omega_0} + \frac{2}{\pi N}$$

which shows that losses due to the electron–electron interaction in the well do not prevent the instability. Equations (15) and (17) show that the large value obtained for α^* is due to the large value of the ratio V_0/Δ (figure 3(b)), whereas N should not be much less than 10 for the assumption that oscillations in the well are weakly disturbed by the current to be valid.

The power delivered by an external direct current source to plasma vibrations in PQW can be estimated as $P = \frac{1}{2}(\delta F_{ext})^*\delta\upsilon$, where the superscript $*$ denotes the complex conjugate and $\delta\upsilon = \mathrm{j}\omega\delta x$ is the velocity of the electron collective movement in the well. The amplitude of the PQW's charge displacement in the regime of stable oscillations can be estimated as $\delta x \approx I_0/\alpha$ and, by making use of the expressions $\delta F_{ext} = (2\pi/\varepsilon)Q_0\delta Q$, $\delta Q \simeq I_0/\omega$ and equation (6) we have

$$P \approx \frac{I_0^2}{\alpha^*}\frac{\pi W}{\varepsilon\omega}$$

which gives for $I_0 \approx 10^4$ A cm^{-2} and $W = 10^{-5}$ cm, $P \simeq 50$ W per 1 cm^2 of the active area of the structure.

For optimal coupling of the structure to an appropriate infrared waveguide system one can expect the power level transformed into infrared radiation to be comparable with P. This suggests that the predicted instability can not only be detected experimentally but could also have practical applications.

3. Taking into account the space charge

It is convenient to use the conservation law for the total current

$$\delta I_{tot} = \delta I_c(x) + \frac{\mathrm{j}\omega\varepsilon}{4\pi}\delta E(x) = \text{constant}. \tag{17}$$

Let the current through the cathode barrier (figure 1) be $\delta I_c(x_1) = g\delta E(x_1)$, where the coefficient g is the cathode barrier conductivity. The conductivity current at arbitrary x in the region $x_1 < x < L_1$ is

$$\delta I_c(x) = \delta I_c(x_1)\exp[-\mathrm{j}k(x - x_1)] = g\delta E(x_1)\exp[-\mathrm{j}k(x - x_1)] \tag{18}$$

where $k = \omega/\upsilon_s$. Equating the total current at $x = x_1$ and at arbitrary x in the region $x_1 < x < x_2$ one finds for the electric field

$$\delta E(x) = \frac{4\pi}{\mathrm{j}\omega\varepsilon}\delta E(x_1)\left(\frac{\mathrm{j}\omega\varepsilon}{4\pi} + g - g\,\mathrm{e}^{-\mathrm{j}k(x-x_1)}\right). \tag{19}$$

Integrating $\delta E(x)$ gives the voltage across the $[x_1, x_2]$ region:

$$\delta U_{1,2} = \delta E(x_1)\left[L_1\left(1 + \frac{4\pi g}{\mathrm{j}\omega\varepsilon}\right) + \frac{1}{k}\frac{4\pi g}{\omega\varepsilon}(\mathrm{e}^{-\mathrm{j}\theta_1} - 1)\right] \tag{20}$$

where θ_1 is the transit time $\theta_1 = kL_1 = \omega L_1/\upsilon_s$.

Assuming that the charge density in the region $[x_2, x_3]$ is constant and equal to $(Q_0 + \delta Q)/W$ one obtains for the voltage across this region

$$\delta U_{2,3} = W\delta E(x_2) + \frac{2\pi}{\varepsilon}W\delta Q - \frac{4\pi}{\varepsilon}Q_0\delta x$$

where the displacement of the electron charge distribution δx is given by equation (9) with δE_1 substituted by $\delta E(x_2)$. By using equations (4) and (18) and expressing $\delta E(x_2)$ in terms of $\delta E(x_1)$ with the help of equation (19) one obtains the expression

$$\delta U_{2,3} = W\delta E(x_1)\left[\left(1+\frac{\mathrm{j}\alpha^*\omega_{pl}^2}{2A}-\frac{\omega_{pl}^2}{A}\right)\left(1+\frac{4\pi g}{\mathrm{j}\omega\varepsilon}(1-\mathrm{e}^{\mathrm{j}\theta_1})\right)+\frac{2\pi g}{\mathrm{j}\omega\varepsilon}\,\mathrm{e}^{-\mathrm{j}\theta_1}\right]. \tag{21}$$

To calculate the potential drop in the third region we introduce a parameter ψ

$$\psi = \delta I_c(x_3)/\delta I_{tot}. \tag{22}$$

From equation (17) we find

$$\delta I_{tot} = \left(\frac{\mathrm{j}\omega\varepsilon}{4\pi}+g\right)\delta E(x_1). \tag{23}$$

The current $\delta I_c(x_3)$ can be calculated by using equations (5), (6), (19) and (9) (with δE_1 substituted by $\delta E(x_2)$):

$$\delta I_c(x_3) = \frac{\alpha^*\omega_0 e^2 n_0}{m}\left(1+\frac{4\pi g}{\mathrm{j}\omega\varepsilon}(1-\mathrm{e}^{-\mathrm{j}\theta_1})\right)\frac{\delta E(x_1)}{A}. \tag{24}$$

Equations (22) to (24) give

$$\psi = \frac{\alpha^*\omega_0\omega_{pl}^2}{A}\,\frac{\varepsilon}{4\pi g + j\omega\varepsilon}\left(1+\frac{4\pi g}{j\omega\varepsilon}(1-\mathrm{e}^{-\mathrm{j}\theta_1})\right). \tag{25}$$

From equation (17) one has for an arbitrary x in the interval $[x_3, x_4]$

$$\delta I_{tot} = \frac{\mathrm{j}\omega\varepsilon}{4\pi}\delta E(x) + \psi\,\mathrm{e}^{-\mathrm{j}k(x-x_3)}\delta I_{tot}$$

and therefore

$$\delta E(x) = \frac{4\pi}{\mathrm{j}\omega\varepsilon}\delta I_{tot}(1-\psi\,\mathrm{e}^{-\mathrm{j}k(x-x_3)}). \tag{26}$$

Integrating $\delta E(x)$ gives the voltage across the $[x_3, x_4]$ region:

$$\delta U_{3,4} = \frac{4\pi}{j\omega\varepsilon}\delta I_{tot}\left(L_2 - \mathrm{j}\frac{\psi}{k}(\mathrm{e}^{-\mathrm{j}\theta_2}-1)\right) \qquad \theta_2 = kL_2$$

and using equation (23) we finally have

$$\delta U_{3,4} = \delta E(x_1)\left(1+\frac{4\pi g}{\mathrm{j}\omega\varepsilon}\right)\left(L_2 - \mathrm{j}\frac{\psi}{k}(\mathrm{e}^{-\mathrm{j}\theta_2}-1)\right). \tag{27}$$

The potential difference between the cathode and the anode is $\delta U = \delta U_{1,2} + \delta U_{2,3} + \delta U_{3,4}$. The stability of the system is determined by the boundary condition $\delta U = 0$ together with equations (20), (21) and (27).

As a result one obtains an equation of the type

$$0 = A + O\left(\frac{L_2}{L_1}\right) + O\left(\frac{W}{L_1}\right) + O\left(\frac{1}{kL_1}\right)$$

where the three last terms are proportional to L_2/L_1, W/L_1 and $1/(kL_1)$, respectively. The ratio $1/(kL_1)$ is about 10^{-2} for $\upsilon_s \times 10^7$ cm s^{-1}, $\omega_0 = 2\pi\times 2\times 10^{12}$ s^{-1} and $L_1 = 1\ \mu$m. Therefore if $L_1 \gg L_2, W$ accounting for the space charge does not change the instability threshold.

The space-charge effects become important when L_1 and L_2 are of the same order of magnitude. It can be shown that, by proper choice of L_1 and L_2, one can suppress the plasma instability and at the same time the differential resistance of the structure can be negative within some frequency range if condition (13) is fulfilled. This can be used for the amplification of infrared radiation. The additional (to the simple consideration) effect which is brought about by the space charge is the well-known 'transit time' effect. When there is a non-zero phase shift between the conductivity current entering some layer and the total current then the finite time it takes the charge to cross the layer can result in a negative differential resistance. This effect is widely used in solid state microwave transit time devices. In the present structure the phase shift between the conductivity and total currents in the region $[x_3, x_4]$ strongly depends on frequency because the phase of the plasma vibrations is a rather complicated function of the frequency. It is tempting to use the transit time effect together with the plasmon-assisted tunnelling for the amplification of infrared radiation. The above expressions allow one to analyse the structure operation for arbitrary L_1, L_2 and frequency.

4. Conclusion

We have shown that the electron layer in a PQW can be made unstable with respect to sloshing plasma oscillations. It appears that the threshold condition can be attained at low (4 K) and probably at liquid nitrogen temperatures because the electron mobility could still be rather high at this temperature. Considering the discussed structure as a possible candidate for a solid-state far-infrared source, it is worth noticing that the structure allows for some degree of frequency-tunability. The external potential which defines the well (figure 1) can be tailored in a very precise way during the growth in order to introduce non-parabolic terms, as discussed in [20]. The resonant plasma frequency in such a well depends slightly on the electron density in the well and therefore on the structure bias conditions. We also believe that sloshing-plasma-mode-assisted tunnelling, the process which involves interaction between collective and single-particle excitations, is a very interesting subject in its own right, and can provide an additional way of studying the electronic structure of wide parabolic quantum wells.

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